

On the Dalitz Plot Approach in Non-leptonic Charm Meson Decays

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Abstract

We claim that the non-resonant contribution to non-leptonic charm meson decays may not be constant in the phase space of the reaction. We argue that this can be relevant for any weak reaction. We discuss in detail the decay $D^+ \rightarrow K^- \pi^+ \pi^+$.

Non-leptonic charm meson decays have been extensively studied both theoretically and experimentally. The high diversity and low multiplicity of decay channels provide important information on both weak and strong interactions. These decays have contributions from resonances in intermediate states, as well from the direct non-resonant (NR) decay. The understanding of the decay pattern of charm mesons as a whole, and therefore the extraction of the decay partial widths for all contributing states, is essential in addressing many open problems in charm physics.

The Dalitz plot analysis [1] is a powerful technique widely used in the study of resonance substructures on charmed meson decays. The plot represents the phase space of the decay and it is weighted by the squared amplitude

of the reaction. Therefore, it contains information on both the kinematics and the dynamics. Within this technique, intermediate resonant and non-resonant contributions are fitted to get the respective amplitudes and phases. The corresponding partial decay widths can then be obtained.

When experimental data on non-leptonic decays of charm mesons became available in the seventies, J. Wiss *et al* [2] used the Dalitz plot technique to search for the spin of the recently discovered charged D meson. They found a result statistically compatible with a flat distribution. Assuming that the structure on the Dalitz plot is dominated by the hadronic spin amplitude [3], they concluded the D^+ meson would be a spin 0 particle.

Subsequently, resonances were found in higher statistics experiments. Since then, attention has focused on them and the NR contribution has been assumed to be constant. For instance, data on non-leptonic decays of the D meson has been fitted [4, 5, 6, 7, 8] using Breit-Wigner functions[9] to represent the various resonances (with the respective angular distribution) and a constant function to describe the NR contribution [10].

Although the above parameterization is widely used, a very poor fit has been reported [5, 8], suggesting that it may not be adequate to describe these decays. These poor results do not improve with higher statistics or considering a larger number of resonances[8]. Moreover, this problem appears in all the $D \rightarrow K\pi\pi$ decay channels already measured ($D^0 \rightarrow \bar{K}^0\pi^+\pi^-$, $D^+ \rightarrow \bar{K}^0\pi^+\pi^0$, $D^+ \rightarrow K^-\pi^+\pi^+$ and $D^0 \rightarrow K^-\pi^+\pi^0$) [11] and the worst fit is obtained for $D^+ \rightarrow K^-\pi^+\pi^+$, where the NR contribution dominates[8]. (In this case, with 29 degrees of freedom, the χ^2 per degree of freedom is as bad as 3.01.)

A possible explanation for these discrepancies is the incorrect use of a constant amplitude for the NR contribution. An incorrect parameterization will certainly influence the fit of the resonances and consequently the extracted values of amplitudes and phases. As an example, MarkIII reported[5] significant discrepancies on the measurement of the branching ratio (BR) of $D^+ \rightarrow \bar{K}^*\pi^+$ obtained from the different final states $K^0\pi^0\pi^+$ and $K^-\pi^+\pi^+$. Note that while the NR contribution to the first final state is of the order of 15% of the total partial decay width, in the second it is as large as 80%.

Here, we claim that NR charm meson decays may contain information beyond the simple hadronic amplitude of a spin zero particle decaying into three spin zero daughters. Since we are dealing with weak decays, signatures of this fundamental interaction can directly appear in the NR amplitude.

In weak interactions between quarks and leptons helicity plays an important role. Consequently, one expects a significant dependence of the weak amplitudes on the momenta of the interacting particles. Thus, the dynamics of these reactions vary from point to point of the phase space and the significance of this variation depends on the specific physical reaction.

This should be particularly important in weak decays of charm mesons. The large value of the charm quark mass allows for a quasi perturbative treatment of QCD. Furthermore, charm quark decays into light quarks and this enhances the importance of helicity. For example, we can see the effect of weak partonic mechanism responsible for the Cabibbo favored D meson decays, i.e. $c \rightarrow s\bar{u}$, by analysing the decay of τ leptons, $\tau \rightarrow \mu\bar{\nu}_\mu\nu_\tau$, which are essentially similar. This simple example will shed some light on the dependence of a weak reaction on its phase space.

The theoretical Dalitz plot corresponding to the decay $\tau \rightarrow \mu\bar{\nu}_\mu\nu_\tau$ can be obtained by taking the well known decay amplitude of pure leptonic decays[12]. This decay amplitude can be written as a function of two invariant variables defining a Dalitz plot, e.g., $m_{\mu\bar{\nu}_\mu}^2 \equiv (p_\mu + p_{\bar{\nu}_\mu})^2$ and $m_{\mu\nu_\tau}^2 \equiv (p_\mu + p_{\nu_\tau})^2$ to give

$$|\mathcal{M}_{\tau \rightarrow \mu\bar{\nu}_\mu\nu_\tau}|^2 \propto m_{\mu\nu_\tau}^2 (m_\tau^2 - m_{\mu\nu_\tau}^2) \quad (1)$$

where m_τ is the τ mass.

The dynamics of the reaction has a quadratic dependence on the variable $m_{\mu\nu_\tau}^2$. As the Dalitz plot is weighted by $|\mathcal{M}_{\tau \rightarrow \mu\bar{\nu}_\mu\nu_\tau}|^2$, equation (1) shows that a Dalitz plot of a pure weak decay has indeed significant variations along the phase space.

Obviously, due to the hadronization process of the partons after their weak interaction, the result of the previous example cannot be simply translated into hadronic decays. In the latter case, one has to take into account non-perturbative QCD effects involved in the final hadronic state formation. In order to make an estimate of the effect of the dynamics in the Dalitz plot, we use an approximate method to describe hadronic decays. The method is based on both the factorization technique [13] and an effective Hamiltonian [14, 15] for the partonic interaction and has been successfully used to describe heavy meson decays[13, 16].

As we are interested in the NR contributions, we analyse the channel $D^+ \rightarrow K^-\pi^+\pi^+$, which has a very large NR branching ratio, as mentioned

above. The effective Hamiltonian for the weak vertex $c \rightarrow s\bar{u}\bar{d}$ is [14, 15]:

$$\mathcal{H}_{eff} = \left(\frac{G_F}{\sqrt{2}}\right) \cos^2 \theta_c [a_1 : (\bar{s}c)(\bar{u}d) : + a_2 : (\bar{s}d)(\bar{u}c) :] \quad (2)$$

where $(\bar{q}q')$ is a short-hand notation for $\bar{q}\gamma^\mu(1-\gamma_5)q'$. The coefficients a_1 and a_2 characterize the contribution of the effective charged and neutral currents respectively, which include short-distance QCD effects. Their values have been fitted in the case of charm meson two-body decays (see for example reference [14]). The diagrams contributing to the decay $D^+ \rightarrow K^-\pi^+\pi^+$ are shown in Figure (). Using factorization we obtain the following decomposition for the hadronic amplitude

$$\begin{aligned} \mathcal{M}_{D^+ \rightarrow K^-\pi^+\pi^+} &= \left(\frac{G_F}{\sqrt{2}}\right) \cos^2 \theta_c [a_1 \langle K^-\pi_1^+ | \bar{s}c | D^+ \rangle \langle \pi_2^+ | \bar{u}d | 0 \rangle \\ &\quad + a_2 \langle K^-\pi_1^+ | \bar{s}d | 0 \rangle \langle \pi_2^+ | \bar{u}c | D^+ \rangle + (\pi_1^+ \leftrightarrow \pi_2^+)] . \end{aligned} \quad (3)$$

Let us first discuss the term driven by a_1 , i.e, the one of Figure (.a). The most general form to decompose the first matrix element can be written in terms of four form factors[17]. Using the parameterization of reference [18], we can write:

$$\langle K^-\pi_1^+ | \bar{s}c | D^+ \rangle = A_1^\mu F_1 + A_2^\mu F_2 + iV_3^\mu F_3 + A_4^\mu F_4 , \quad (4)$$

where

$$\begin{aligned} A_1^\mu &= p_K^\mu + p_D^\mu - Q^\mu \frac{Q \cdot (p_K + p_D)}{Q^2} , \\ A_2^\mu &= p_{\pi_1}^\mu + p_D^\mu - Q^\mu \frac{Q \cdot (p_{\pi_1} + p_D)}{Q^2} , \\ V_3^\mu &= \epsilon^{\mu\alpha\beta\gamma} p_K^\alpha p_{\pi_1}^\beta p_D^\gamma , \\ A_4^\mu &= Q^\mu = p_K^\mu + p_{\pi_1}^\mu - p_D^\mu = -p_{\pi_2}^\mu . \end{aligned}$$

The terms proportional to F_1 , F_2 and F_4 originate from the axial vector part of the matrix element whereas the one proportional to F_3 originates from the vector part; the terms proportional to F_1 , F_2 and F_3 correspond to spin 1 and F_4 to spin 0. The four form factors depend on three variables $m_1^2 = (p_k + p_{\pi_1})^2$, $m_2^2 = (p_k + p_{\pi_2})^2$ and Q^2 which is a constant (m_π^2) in this case.

The second matrix element in equation (3) has the well known form

$$\langle \pi_2^+ | \bar{u}d | 0 \rangle = i f_\pi p_{\pi_2}^\mu . \quad (5)$$

The only contributing term in equation (4) after multiplying it by equation (5), is the axial spin 0 term, i.e.,

$$\langle K^- \pi_1^+ | \bar{s}c | D^+ \rangle \langle \pi_2^+ | \bar{u}d | 0 \rangle = (p_{\pi_2} \cdot p_{\pi_1}) (i f_\pi p_{\pi_2}^\mu) = i f_\pi m_\pi^2 F_4 . \quad (6)$$

To find the contribution of Figure (.b), one can use the well known expressions[19]

$$\begin{aligned} \langle \pi_2^+ | \bar{u}c | D^+ \rangle &= \left[(p_D + p_{\pi_2})^\mu - \frac{m_D^2 - m_\pi^2}{q^2} (p_D - p_{\pi_2})^\mu \right] F_{D\pi}^{1-}(q^2) \\ &+ \frac{m_D^2 - m_\pi^2}{q^2} (p_D - p_{\pi_2})^\mu F_{D\pi}^{0+}(q^2) \end{aligned}$$

and

$$\begin{aligned} \langle K^-(p_K) \pi_1^+ | \bar{s}d | 0 \rangle &= \langle \pi_1^+ | \bar{s}d | K^+(-p_K) \rangle = \\ &\left[(-p_K + p_{\pi_1})^\mu - \frac{m_K^2 - m_\pi^2}{q^2} (-p_K - p_{\pi_1})^\mu \right] f_+(q^2) \\ &+ \frac{m_K^2 - m_\pi^2}{q^2} (-p_K - p_{\pi_1})^\mu f_0(q^2) . \end{aligned}$$

In the equations above, $q^2 = (p_D - p_{\pi_2})^2 = (-p_K - p_{\pi_1})^2$ while the functions $F_{D\pi}^{J^P}(q^2)$ (corresponding to a current of spin parity J^P), $f_+(q^2)$ and $f_0(q^2)$ are form factors. We return to them later.

We then find for the second contribution in equation (3),

$$\begin{aligned} \langle \pi^+ | \bar{u}c | D^+ \rangle \langle K^- \pi^+ | \bar{s}d | 0 \rangle &= F_{D\pi}^{1-}(m_1^2) f_+(m_1^2) (m_D^2 + m_K^2 + 2m_\pi^2 - 2m_2^2 - m_1^2) \\ &+ [F_{D\pi}^{1-}(m_1^2) f_+(m_1^2) - F_{D\pi}^{0+}(m_1^2) f_0(m_1^2)] \frac{(m_D^2 - m_\pi^2)(m_K^2 - m_\pi^2)}{m_1^2} \\ &+ (m_1^2 \leftrightarrow m_2^2) \end{aligned} \quad (7)$$

where we have explicitly introduced the Dalitz plot variables m_1^2 and m_2^2 defined above.

The contribution of diagram (.a), given by equation (6) is proportional to $f_\pi m_\pi^2$. Thus, unless the form factor F_4 is unacceptably large ($F_4 \sim 10^3$), we can safely neglect this contribution in favor of that of diagram (.b), given by equation (7) which contains m_D^2 . As an aside, it is possible that the NR part of the decay $D^+ \rightarrow K^- \pi^+ \pi^+$ is large precisely because the contribution of diagram (.b) is *not* small.

The NR contribution to the amplitude of the decay $D^+ \rightarrow K^- \pi^+ \pi^+$ can thus be simply written replacing equation (7) in (3), neglecting the contribution of Figure (.a). The final expression thus depends on the effective coefficient a_2 and the four form factors. The two $D\pi$ form factors $F_{D\pi}^{JP}(q^2)$, have well established expressions[15] :

$$F_{D\pi}^{JP}(q^2) = \left(1 - \frac{q^2}{M_{D\pi,JP}^2}\right)^{-1} \quad (8)$$

where $M_{D\pi,1^-} = 2.01$ GeV and $M_{D\pi,0^+} = 2.2$ GeV. They have been successfully used in the kinematic range we are considering here. The poles lie outside our kinematic region. The $K\pi$ form factors, $f_+(q^2)$ and $f_0(q^2)$, can be extracted from the semi-leptonic decays $K \rightarrow \pi l \nu$, with $l = e, \mu$. Nevertheless, it is not clear that the usual parameterization[12]

$$f_+(q^2) = f_+(0) \left(1 + \lambda_+ \frac{q^2}{m_\pi^2}\right), \quad f_0(q^2) = f_0(0) \left(1 + \lambda_0 \frac{q^2}{m_\pi^2}\right) \quad (9)$$

is valid in the whole kinematic region of our reaction. In equation (9), $f_+(0) = f_0(0) = 1$ and the other coefficients have been measured to be[20]: $\lambda_+ \approx 0.03$ independent of the measured channel, whereas the value of λ_0 depends on the decay: $\lambda_0 \approx 0$ for $K^- \rightarrow \pi^0 \mu^- \nu$ and $\lambda_0 \approx 0.025$ for $K^0 \rightarrow \pi^+ \mu^- \nu$.

In order to check the validity of this calculation scheme, we have evaluated the NR partial decay width $\Gamma(D^+ \rightarrow K^- \pi^+ \pi^+)_{NR}$ using the expressions above. With $\lambda_0 = 0$ and the value of a_2 extracted from two body decay[14], we find a branching ratio (BR) of 9% which is close to the reported experimental value[20] $7.3 \pm 1.4\%$ obtained by fitting the NR contribution to a constant. We studied the stability of this result under the change of the parameters λ_+ and λ_0 : if we take the various values extracted from different channels we find that the BR varies less than 30%. Even assuming constant form factors ($\lambda_+ = \lambda_0 = 0$), the BR remains of the same order of magnitude.

Figure () shows the Dalitz plot for the NR contribution to the decay $D^+ \rightarrow K^- \pi^+ \pi^+$ as a function of the variables m_1^2 and m_2^2 . It has been generated by Monte Carlo simulation with a weight proportional to the square of the amplitude in equation (3), using equation (7). We have considered the same central value of the parameters as above. As one can see from equation (7) and Figure (), according to this calculation the matrix element describing the dynamics of the NR contribution to the decay $D^+ \rightarrow K^- \pi^+ \pi^+$ significantly varies along the phase space of the reaction. Its shape remains almost the same for other values of the parameters of the $K\pi$ form factor. This is still valid even if we take the four form factors as constants.

However, the result presented in Figure () has been obtained using an approximate method. Non-perturbative effects, present in this decay through the exchange of soft gluons, or final state interactions could change the structure shown in this figure. In the extreme case where non-perturbative effects completely dominate the decay, the structure will be washed out because of the dispersive nature of these effects, therefore obtaining the flat contribution predicted by the pure hadronic decay of a zero spin particle. Comparison between the distribution shown in Figure () and experimental data will thus be a test for the validity of the factorization method.

In summary, we have shown that the natural parameterization for the non-resonant part of charm decays – based in the spin amplitude of the hadronic decay – could significantly change due to the fundamental weak interaction between quarks. The appearance of these structures in the plot could be responsible for the problems of the various experimental teams with the convergence of their fits. To clarify this point, it is important in future analyses to use a parameterization for the non-resonant contribution going beyond the simple constant.

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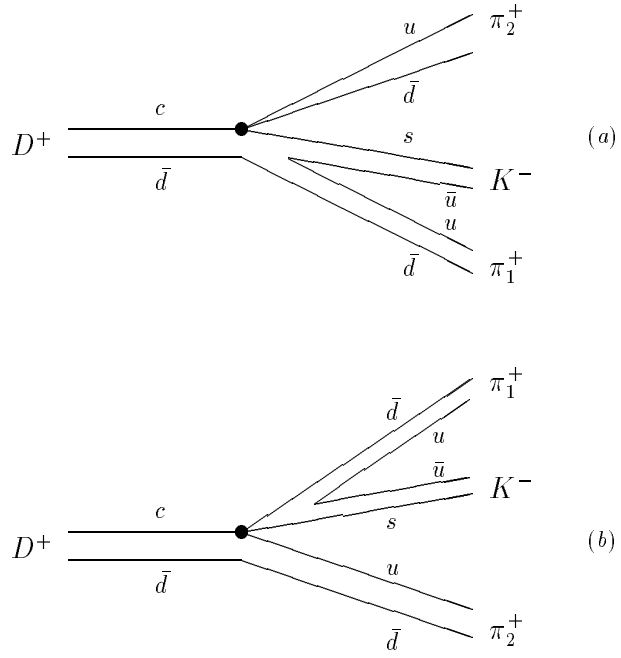


Figure 1: The two diagrams contributing to the decay $D^+ \rightarrow K^- \pi^+ \pi^+$ according to the effective Hamiltonian of equation (2).

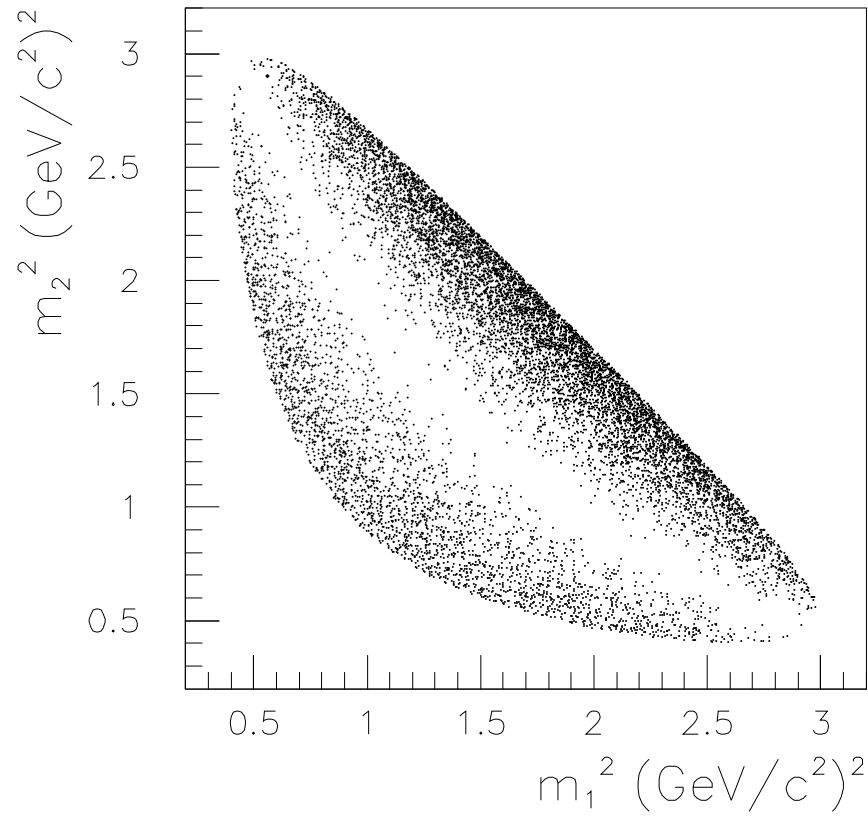


Figure 2: The Dalitz plot of the decay $D^+ \rightarrow K^- \pi^+ \pi^+$, weighted by $|\mathcal{M}_{D^+ \rightarrow K^- \pi^+ \pi^+}|^2$ as in equations (3) and (7), generated via Monte Carlo. The Dalitz plot variables are $m_1^2 \equiv (p_K + p_{\pi_1})^2$ and $m_2^2 \equiv (p_K + p_{\pi_2})^2$.